Activity description

Students solve differential equations to find functions to model the value of a car in terms of its age. Then they compare their results with real data.

Suitability

Level 3 (Advanced)

Time

1–2 hours

Resources

Student information sheet, spreadsheet
Optional: slideshow

Equipment

Access to car prices (paper versions of trade magazines or the internet)
Optional: graphic calculators, internet access

Key mathematical language

Gradient, differential equation, integration, constant of integration, model, function

Notes on the activity

Students will need to know how to solve differential equations involving power, exponential and logarithmic functions.

Students will also need to be able to sketch the related graphs and interpret their key features (such as gradients, intercepts, asymptotes); and compare these with what happens in reality.

The slideshow can introduce the activity. The slides give the data and a graph of value against age for a Toyota Aygo hatchback, then a list of suggested models.

After class discussion, tell students how many models they should study in depth, and whether they should work individually or in pairs or groups.

There are several trade guides giving information about how the value of cars falls with time. Some data is also given in the spreadsheet, 'FSMA What's it worth data for students 2011', but this data will go out of date very quickly and you may need to update it before use.

Most students will probably use real data to evaluate the initial value, V_0, and the constant of proportionality, k, for a particular car (or cars) and hence find particular solutions.

More able students should be encouraged to find more general solutions like those given below, and then use the data they collect to evaluate these solutions.

The differential equation, its solution and a sketch graph are given below for each of the models. In all cases, £V represents the value of the car at age t years, V_0 denotes the initial value of the car, and k is a constant.

It would be useful to conclude the activity with a class discussion in which students present and discuss their findings.

During this discussion you could use the spreadsheet 'FSMA What's it worth data for teachers 2011'. This gives the data and graph for the Toyota Aygo hatchback.

You may find it useful to use Excel to draw trendlines for comparison with the functions students have found.

Points for discussion

How the rate of depreciation appears on a graph of value against time.

Whether the gradient is positive or negative.

How calculus can be used to find functions to model the value in terms of age.

How to find the constants of integration from the given data to get particular solutions.

Extensions

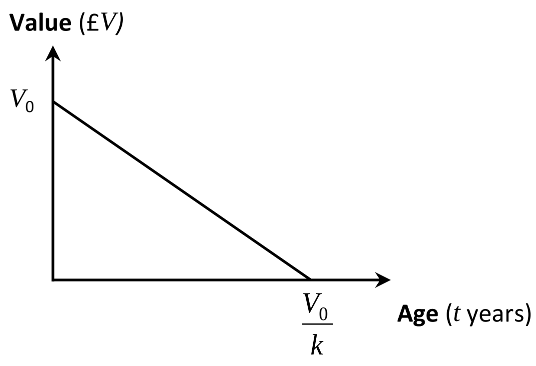
More able students could be encouraged to find more general solutions like those given below. Then use the data they collect to evaluate these solutions.

Answers

Model A Value of car depreciates at a constant rate

Differential equation: $\frac{dV}{dt} = -k$

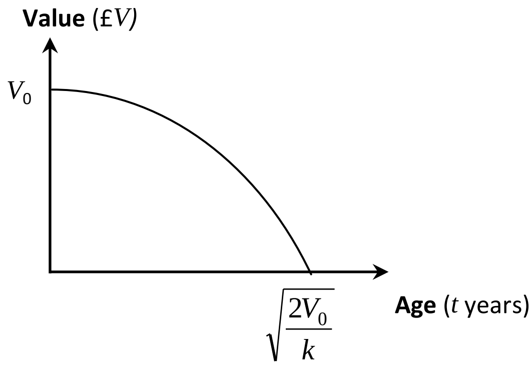
Solution: $V = V_0 - kt$



Model B Rate of depreciation is proportional to age of the car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -kt$

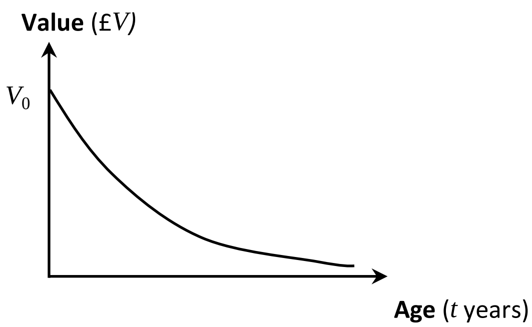
Solution: $V = V_0 - \frac{1}{2}kt^2$



Model C Rate of depreciation is proportional to value of the car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -kV$

Solution: $V = V_0 e^{-kt}$



Model D Rate of depreciation is inversely proportional to age of car

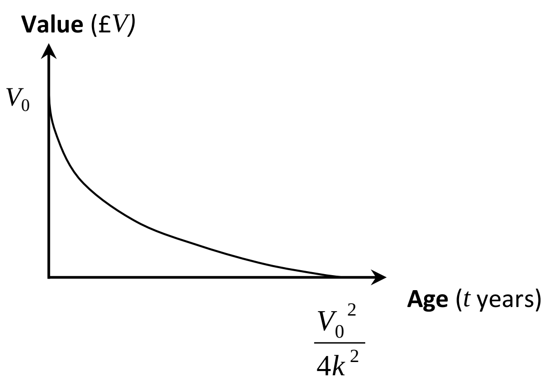
Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{k}{t}$

Solution: $V = -k\ln t + c$ is unsuitable because of the discontinuity at $t = 0$

Model E Rate of depreciation inversely proportional to square root of age of car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{k}{\sqrt{t}}$

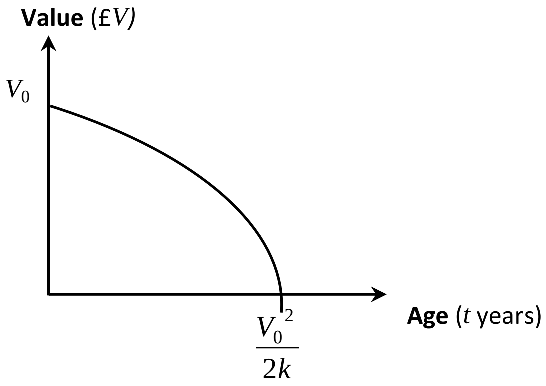
Solution: $V = V_0 - 2k\sqrt{t}$



Model F Rate of depreciation is inversely proportional to value of car

Differential equation: $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -\dfrac{k}{V}$

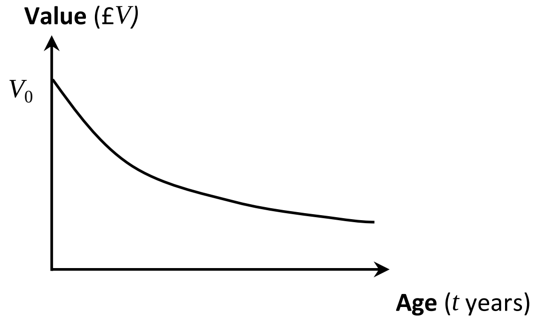
Solution: $V = \sqrt{V_0{}^2 - 2kt}$



Model G Rate of depreciation is proportional to square of value of car

Differential equation: $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -kV^2$

Solution: $V = \dfrac{V_0}{1 + kV_0 t}$



Model H Rate of depreciation is proportional to square root of value of car

Differential equation: $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{V}$

Solution: $V = \left(\sqrt{V_0} - \tfrac{1}{2}kt\right)^2$